**Exam. Code : 103205**
**Subject Code : 1203**

## B.A./B.Sc. 5th Semester
## MATHEMATICS
## Paper—I (Dynamics)

Time Allowed—3 Hours] [Maximum Marks—50

**Note :—** Attempt ***five*** questions in all selecting at least ***two*** from each section.

## SECTION—A

1. (a) A point moving with uniform acceleration in a straight line describes equal distances in time $t_1$, $t_2$, $t_3$; show that $\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{3}{t_1+t_2+t_3}$.

   (b) A, B, C are three points vertically below the point O such that OA = AB = BC. If the particle falls from rest at O, prove that the times of describing OA, AB and BC are as $1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2})$. 5,5

2. Masses P and Q in a Atwood's machine are allowed to move from rest any distance x. If P is greater than Q, show that the mass which must suddenly be removed from P at the end of distance x, so that the motion in the same sense may continue a further distance nx, is

$$\frac{(n+1)(P^2-Q^2)}{(n+1)P+(n-1)Q}.$$ 10

3. (a) Two masses $m_1$, $m_2$ are connected by an inelastic string; $m_2$ is placed on a smooth horizontal table and the string passes over a light smooth pulley at the edge of the table and $m_1$ is hanging freely. Determine the motion and the tension in the string. Find also the pressure on the pulley.

(b) A body sliding down a smooth inclined plane is observed to cover equal distances, each equal to $l$, in consecutive intervals of time $t_1$ and $t_2$. Show that inclination of the plane is

$$\sin^{-1}\left[\frac{2\ell(t_1-t_2)}{gt_1t_2(t_1+t_2)}\right].$$ 5,5

4. (a) A particle starts from rest and moves along a straight line with an acceleration f varying as $t^n$. If v be the velocity at a distance s from the starting point, show that $(n + 1)v^2 = (n + 2)fs$.

(b) A particle free to move along the x-axis is subjected to a force $mF_0\cos pt$ acting along x-axis. At $t = 0$, $x = 0$ and $v = 0$. Show that at any time

$t$, $x = \frac{F_0}{p^2}(1-\cos pt)$. Here m is the mass of the particle. $F_0$ and p are constants. 5,5

5. A particle is performing simple harmonic motion of period T about a centre O and it passes through the position P (OP = b) with velocity v in the direction OP. Prove that the time which elapses before it comes to P is $\frac{T}{\pi}\tan^{-1}\frac{vT}{2\pi b}$. 10

## SECTION—B

6. (a) A particle is projected with velocity u so that its range on a horizontal plane is twice the greatest height attained. Show that range is $\frac{4u^2}{5g}$.

(b) The maximum height of a projectile is h and angle of projection is $\alpha$. Find out the difference of time when it is at height of $h \sin^2\alpha$. 5,5

7. A particle is projected from O at an elevation $\alpha$ and after time t, the particle is at P. Prove that $\tan \beta = \frac{1}{2}(\tan \alpha + \tan \theta)$ where $\beta$ and $\theta$ are respectively the inclinations to the horizontal of OP and of the direction of motion of the particle when at P. 10

8. (a) A train of mass M kg is ascending a smooth incline of 1 in n and when the velocity of the train is vm/sec, its acceleration is f m/sec$^2$. Prove that the effective power of the engine is $\frac{Mv(nf+g)}{n}$ watts.

(b) Prove that the kinetic energy of a particle of mass m moving with a magnitude of velocity v is $\frac{1}{2}mv^2$.

5,5

9. A particle of mass m is tied to the middle point of an elastic string of natural length $2\,l$ and modulus $\lambda$. The ends of the string are tied to two points on a smooth horizontal table distant 2L ($L > l$). Find the period of small oscillation (i) along the string (ii) perpendicular to the string. 10

10. A pendulum of length $l$ hangs against a wall inclined at an angle $\alpha$ to the horizontal. Show that the time of complete oscillation is $2\pi\sqrt{\dfrac{\ell}{g \sin\alpha}}$. 10